Parity Invariance and Effective Light-Front Hamiltonians

M. Burkardt
Department of Physics
New Mexico State University
Las Cruces, NM 88003-0001
U.S.A.

In the light-front form of field theory, boost invariance is a manifest symmetry. On the downside, parity and rotational invariance are not manifest, leaving the possibility that approximations or incorrect renormalization might lead to violations of these symmetries for physical observables. In this paper, it is discussed how one can turn this deficiency into an advantage and utilize parity violations (or the absence thereof) in practice for constraining effective light-front Hamiltonians. More precisely, we will identify observables that are both sensitive to parity violations and easily calculable numerically in a non-perturbative framework and we will use these observables to constrain the finite part of non-covariant counter-terms in effective light-front Hamiltonians.

I. INTRODUCTION

Light-Front (LF) quantization is very similar to canonical equal time (ET) quantization [1] (here we closely follow Ref. [2]). Both are Hamiltonian formulations of field theory, where one specifies the fields on a particular initial surface. The evolution of the fields off the initial surface is determined by the Lagrangian equations of motion. The main difference is the choice of the initial surface, $x^0 = 0$ for ET and $x^+ = 0$ for the LF respectively. In both frameworks states are expanded in terms of fields (and their derivatives) on this surface. Therefore, the same physical state may have very different wave functions[1] in the ET and LF approaches because fields at $x^0 = 0$ provide a different basis for expanding a state than fields at $x^+ = 0$. The reason is that the microscopic degrees of freedom — field amplitudes at $x^0 = 0$ versus field amplitudes at $x^+ = 0$ — are in general quite different from each other in the two formalisms.

From the purely theoretical point of view, various advantages of LF quantization derive from properties of the ten generators of the Poincaré group (translations P^μ, rotations $\vec{L}$ and boosts $\vec{K}$) [1,2]. Those generators which leave the initial surface invariant ($\vec{P}$ and $\vec{L}$ for ET and P_-, $\vec{P}_\perp$, L_3 and $\vec{K}$ for LF) are "simple" in the sense that they have very simple representations in terms of the fields (typically just sums of single particle operators). The other generators, which include the "Hamiltonians" (P_0, which is conjugate to x^0 in ET and P_+, which is conjugate to the LF-time x^+ in LF quantization) contain interactions among the fields and are typically very complicated. Generators which leave the initial surface invariant are also called *kinematic* generators, while the others are called *dynamic* generators. Obviously it is advantageous to have as many of the ten generators kinematic as possible. There are seven kinematic generators on the LF but only six in ET quantization.

The fact that P_-, the generator of x^- translations, is kinematic (obviously it leaves $x^+ = 0$ invariant!) and positive has striking consequences for the LF vacuum [2]. For free fields $p^2 = m^2$ implies for the LF energy $p_+ = \left(m^2 + \vec{p}_\perp\right)/2p_-$. Hence positive energy excitations have positive p_-. After the usual re-interpretation of the negative energy states this implies that p_- for a single particle is non-negative [which makes sense, considering that $p_- = \left(p_0 - p_3\right)/\sqrt{2}$]. P_- being kinematic means that it is given by the sum of single particle momenta p_-. Combined with the non-negativity of p_- this implies that, even in the presence of interactions, the physical vacuum (ground state of the theory) differs from the Fock vacuum (no particle excitations) only by so-called *zero-mode* excitations, i.e. by excitations of modes which are independent of the longitudinal LF-space coordinate x^-. Due to this simplified vacuum structure, the LF-framework seems to be the only framework, where a constituent quark picture in a strongly interacting relativistic field theory has a chance to make sense [3–6].

Whenever the generator of a symmetry is dynamical (contains interactions) it is somewhere between very difficult and impossible to monitor and maintain that symmetry at each step of a calculation — unless of course on can solve the theory exactly. A typical example is the boost invariance, which, in the context of equal time quantization, is generated by a dynamical operator. It is thus *not manifestly* true that $E_n^2 = m_n^2 + \vec{p}^2$, i.e. the eigenvalues satisfy the correct dispersion relation if and only if one starts from the correct renormalized Hamiltonian, with the right counter-terms (for the regulators employed). Now suppose, one does not know the Hamiltonian precisely but has some idea how it may look like: for example one knows the operators that appear in the Hamiltonian but not their coefficients. In such a situa-

[1] By "wave function" we mean here the collection of all Fock space amplitudes.

tion it should, at least in principle,[2] be possible to use relativistic covariance as a renormalization condition that can be used to pin down some of the renormalization constants in the Hamiltonian[3].

In Hamiltonian LF calculations one faces a very similar problem: in practical non-perturbative calculations one often leaves out degrees of freedom, such as zero-modes and other high-energy degrees of freedom [3]. Because of such (in practical non-perturbative calculations nearly unavoidable!) approximations it is in general not guaranteed that one recovers non-manifest symmetries (on the LF: parity and rotational invariance) in the end. On the contrary, without appropriate[4] counter-terms in the Hamiltonian one is almost guaranteed to violate these symmetries.

In this work, an attempt will be made to turn these problems into an advantage. More precisely, we will identify physical observables which are easily accessible in a practical non-perturbative calculation and which are sensitive to violations of covariance.

Parity transformations take $x^+ \longleftrightarrow x^-$, i.e. LF-time and LF-space are interchanged. Within the LF formalism, this is a very complicated transformation: in the above language, parity transformations are obtained by *dynamical* operators because the initial surface ($x^+ = 0$) is not invariant under parity. From the practical point-of-view, this has the following consequences. First, given a LF Hamiltonian, most approximations to that Hamiltonian are likely to break parity invariance. Secondly, even if one does a "perfect numerical job", parity invariance may still be broken because it may have been broken already at the level of regularization and renormalization: most regulators that are practically useful within the LF-formalism break parity invariance. This also includes effects that arise when zero-modes are omitted (or eliminated) as dynamical degrees of freedom [7–15].

The counter-terms introduced in the renormalization procedure are thus not only supposed to cancel the infinities but also to restore parity invariance (in the limit as the cutoff goes away). In general, restoring parity requires an additional finite renormalization! This issue will be the main subject in the rest of this paper.

Many of the general statements made so far also apply to rotational invariance, i.e. at least in principle this paper could also have been written about rotational invariance [16]! However, in practical LF calculations, rotational invariance is usually broken much more badly than parity invariance: for example, in the transverse lattice formulation of LF field theory [17–21], the classical action is still invariant under parity (which maps the 1+1 dimensional sheets onto themselves), but not under general rotations which mix the continuous longitudinal direction and the discrete transverse direction. Thus for a given transverse lattice (with fixed transverse spacing), if one does a perfect numerical job and if one does the renormalization right, one should obtain a perfectly parity invariant theory, whereas, under the same conditions, rotational invariance should only be recovered if one furthermore takes the limit of zero lattice spacing and infinite lattice volume.

The paper is organized as follows. In Section II and III, some observables that are sensitive to violations of parity are identified and we will discuss their usefulness in the context of practical non-perturbative LF-calculations. In Section IV, we will illustrate the formalism by studying the these observables in the context of a concrete example: 1+1 dimensional Yukawa theory. In Section V, we will summarize the findings and discuss potential applications of the formalism to QCD.

II. THE DIFFICULTY IN FINDING SENSITIVE AND SENSIBLE OBSERVABLES

There are of course infinitely many relations between matrix elements one can write down that are potentially sensitive to parity violations. However, most of them are not useful here because of a number of practical considerations: The main limitations arise since

A) certain relations arising from parity invariance are "protected" by some manifest symmetry, or

B) the matrix elements appearing in those relations are incalculable in praxis.

These points can be illustrated by considering a few examples.

A. Protected Relations

Charge conjugation is a manifest symmetry in the LF formalism. Therefore, certain matrix elements that are in principle sensitive to parity could be "protected" by C-parity. For example, if $|n\rangle$ is an eigenstate of parity then its vacuum to meson scalar and pseudoscalar couplings ($\langle 0|\overline{\psi}\psi|n\rangle$ and $\langle 0|\overline{\psi}i\gamma_5\psi|n\rangle$ respectively) cannot both be nonzero for the same state $|n\rangle$. However, the same statement is true for eigenstates of C-parity — irrespective whether $|n\rangle$ is an eigenstate of parity. Thus, no matter

[2] In practice, this example has a serious flaw: Most non-perturbative numerical techniques project most efficiently on the ground state with zero momentum. Energies of excited states and states with nonzero momentum are typically much more difficult to evaluate.

[3] This is of course only possible as long as the energy scale of the approximations involved is much larger than the kinetic energy associated with the momentum p.

[4] Appropriate means here not only the correct infinite part of the counter-term, which can often be obtained on the basis of perturbative arguments, but also the correct finite part of the counter-term.

how badly parity is violated, as long as manifest C-parity is maintained, either $\langle 0|\overline{\psi}\psi|n\rangle$ or $\langle 0|\overline{\psi}i\gamma_5\psi|n\rangle$ (or both) will always vanish and one cannot exploit these vacuum to meson matrix elements to investigate whether or not parity is violated.

The situation changes when one introduces different quark flavors and considers states that have net flavor and hence are not eigenstates of C-parity, such as $\overline{s}u$[5]. However, even in a multi-flavor theory, these vacuum to meson matrix elements are not very useful in practice for investigations of parity invariance because of some nontrivial operator renormalization issues, which we will discuss below in the context of j^-.

B. Incalculable Matrix Elements

Other selection rules and relations can be derived from the Lorentz transformation properties of vector currents in a parity invariant theory. For example, for the vacuum to meson matrix element of a vector/pseudovector one obtains
3+1 dimensions:

$$\begin{aligned}\langle 0|\overline{\psi}\gamma_\mu\psi|n,s,p\rangle &= s_\mu c_n \\ \langle 0|\overline{\psi}\gamma_\mu\gamma_5\psi|n,s,p\rangle &= p_\mu c_n\end{aligned} \tag{2.1}$$

1+1 dimensions:[6]

$$\begin{aligned}\langle 0|\overline{\psi}\gamma_\mu\psi|n,p\rangle &= \varepsilon_{\mu\nu}p^\nu c_n \\ \langle 0|\overline{\psi}\gamma_\mu\gamma_5\psi|n,p\rangle &= p_\mu c_n,\end{aligned} \tag{2.2}$$

where p_μ, s_μ are the momentum and spin vector respectively. Note that there is no spin in 1+1 dimensions. $\varepsilon_{\mu\nu}$ is the antisymmetric tensor in 1+1 dimensions.

Naively one may think that these relations are useful to detect violations of parity invariance. For example,one can calculate both $c_n^{(+)} \equiv \langle 0|\overline{\psi}\gamma^+\gamma_5\psi|n,p\rangle/p^+$ and $c_n^{(-)} \equiv \langle 0|\overline{\psi}\gamma^-\gamma_5\psi|n,p\rangle/p^-$ independently and then compare the results: in a parity invariant theory the results for c_n extracted from the two Lorentz components of the current should be the same. There are many such relations that one can derive for matrix elements and coupling constants calculated from different Lorentz components. For the purpose of detecting violations of parity invariance in the Hamiltonian LF formalism they are all totally useless!

The flaw in all these examples is that at least one of the Lorentz components of the currents involved in such relations contains a *bad current*: in the LF formalism one usually decomposes the fermion field into dynamical and non-dynamical components

$$\psi = \psi_{(+)} + \psi_{(-)}, \tag{2.3}$$

where $\psi_{(\pm)} \equiv \frac{1}{2}\gamma^\mp\gamma^\pm\psi$. The point is that the (LF-) time derivative of $\psi_{(-)}$ does not enter the Lagrangian and thus $\psi_{(-)}$ satisfies a constraint equation and is usually eliminated by solving this constraint equation.[7] An explicit example will be given in section IV. Since the solution to these constraint equations are typically *nonlinear* expressions (in term of the dynamical degrees of freedom), any operator containing $\psi_{(-)}$ naturally ends up being highly nonlinear when expressed in terms of $\psi_{(+)}$ and the other (dynamical) fields involved in the interactions. For example, in a gauge theory ($A^+ = 0$ gauge) or Yukawa theory, $\psi_{(-)}$ contains a product of $\psi_{(+)}$ and the boson field. These nonlinear terms generally lead to nasty divergences in composite operators involving $\psi_{(-)}$, which is the reason why composite operators involving $\psi_{(-)}$ are called *bad currents*.[8] However, the motivation for this terminology is not only the occurrence of divergences — after all we have become used to divergences in quantum field theory and we have learned how to renormalize the infinities by adding counter-terms — but the fact that the finite part of the counter-term remains a priory ambiguous in this procedure. The bottom line is the following: the goal of this paper is to find ways to use space-time symmetries to constrain the finite parts of the non-covariant counter-terms in the LF-Hamiltonian. Matrix elements of bad currents contain unknown finite renormalizations themselves. In a sense, by considering matrix elements of bad currents we have increased not only the number of equations (renormalization conditions) but also the number of "unknowns" and the net result is questionable. [9]

After these sobering insights about formal limitations in studying parity (and also rotational invariance) violations within the LF framework, let us now turn to practi-

[5]Of course, for a flavor symmetric theory G-parity takes the role of C-parity in this case, but we will assume $m_u \neq m_s$ in this example.

[6]Even if one is only interested in 3+1 dimensional theories, it is useful to consider the 1+1 dimensional relations because the 3+1 dimensional relations assume rotational invariance and are thus likely to be broken. Furthermore the transverse lattice formulation of 3+1 dimensional field theories explicitly utilizes 1+1 dimensional degrees of freedom to approximate the 3+1 dimensional continuum.

[7]This procedure resembles very much the elimination of the 'Coulomb component' of the photon field in Coulomb gauge.

[8]Sometimes one refers to operator which are bilinear in $\psi_{(-)}$, such as $j^- = \psi^\dagger_{(-)}\psi_{(-)}$, as *very bad* operators.

[9]Certain bad currents also enter the Hamiltonian and thus studying their matrix elements does not increase the number of unknowns because these operators have to be renormalized anyway in order to construct the Hamiltonian. However, we will not exploit this fact here any further. See Ref. [22] for some examples where bad currents acquire additional renormalizations.

cal limitations: the most powerful numerical techniques for non-perturbative LF-calculations, the Lanzcos algorithm [23] and Monte Carlo techniques [24] are only good for ground and low lying states (for given good quantum numbers). Excited states and scattering states are very difficult to handle and analyze. This implies that we *cannot* (for practical reasons) use selection rules in the decay of resonances to study parity violations either!

III. PARITY SENSITIVE OBSERVABLES THAT ARE SENSIBLE

The combination of these two restrictions, exclusion of bad currents and ground state (for given good quantum numbers) properties only, severely constrains the possibilities for studying parity violations on the LF in a non-perturbative framework, which is probably why this subject has so far not been studied in more detail.

Fortunately, despite these limitations, there are a few observables left which are are not excluded from the start: Our first example is the inelastic electro-magnetic form factor. Current conservation, i.e. $q_\mu j^\mu = 0$ and parity invariance imply that it should be possible to write the matrix elements of the current operator in the form

$$\langle m, p'|j^\mu(0)|n,p\rangle = \begin{cases} (p^\mu + p'^\mu)\, F_{mn}(q^2) & \text{same parity} \\ q^\mu F_{mn}(q^2) & \text{opposite parity,} \end{cases} \tag{3.1}$$

where $q = p - p'$. For the "good" component, this implies

$$\langle m, p'|j^+(0)|n,p\rangle = \begin{cases} (p^+ + p'^+)\, F_{mn}(q^2) & \text{same parity} \\ q^+ F_{mn}(q^2) & \text{opposite parity.} \end{cases} \tag{3.2}$$

It is implied that the states have been normalized covariantly. Otherwise one has to multiply by appropriate normalization factors. After factoring out the kinematic coefficient [$(p^+ + p'^+)$ for transitions between states of equal parity and q^+ for parity changing transitions] the form factor should depend on q^2 only, but no longer on $q^+ = p^+ - p'^+$ and $q^- = M_n^2/2p^+ - M_m^2/2p'^+$ separately. Of course if parity is violated then any linear combination of $(p^+ + p'^+)$ and q^+ (with two independent form factors) may occur on the r.h.s. and it is no longer possible to obtain a result that depends on q^2 only by factoring out a kinematic coefficient. Denoting $x = q^+/p^+$, energy conservation implies

$$M_n^2 = \frac{q^2}{x} + \frac{M_m^2}{1-x}, \tag{3.3}$$

i.e. for fixed q^2 one obtains a quadratic equation in x

$$x^2 M_n^2 + x\left(M_m^2 - M_n^2 - q^2\right) + q^2 = 0, \tag{3.4}$$

whose solutions typically come in pairs $x_{1/2}$ (except for $M_n = M_m \pm \sqrt{q^2}$). Physically these two solutions correspond to the two cases where the momentum is transferred to the initial state by "hitting it" from the left or from the right. Of course, in a parity invariant theory one should (up to a kinematic factor) obtain the same form factor from these two values of x. However, in a LF calculation this is in general not manifestly true and one can use the equality of the form factor F_{mn} as extracted from x_1 and x_2 as a condition to test parity invariance. Note that this test only works for $m \neq n$ because for $m = n$ invariance under PT (which is usually manifest in the LF formalism), combined with longitudinal boost invariance, guarantees equality of the two form factors extracted from x_1 and x_2.

When one wants to test whether some LF Hamiltonian gives rise to a parity invariant theory, one can use the following procedure:

- One diagonalizes the Hamiltonian and determines the meson wave functions
- Then one calculates the inelastic transition form factor F_{mn} using Eq.(3.2) as a function of the longitudinal momentum transfer x for two arbitrary meson states m and n[10].
- Then one also calculates the invariant momentum transfer q^2 also as a function of x using Eq.(3.3)
- Finally, one parametrically (parameter x) plots F_{mn} versus q^2. If F_{mn} does not turn out to be a unique function of q^2 then parity is violated

Below the practicality of this procedure will be demonstrated in a concrete example. However, before doing this, we should mention a possible caveat: In order to be able to evaluate the matrix element appearing in Eq.(3.2) one needs two ingredients: the states in some basis and the current operator in the same basis. For the "good" current appearing in Eq.(3.2) we will always assume the canonical form in this paper. There are several reasons for doing this. First, it is the most simple form. Second, since no Tamm-Dancoff approximation will be employed in this paper and since we extrapolate to the limit where there is no cutoff in this paper, there is no reason to believe that the good current should be modified from its canonical form.[11] Third, our numerical results explicitly demonstrate that this leads to a self-consistent procedure. Nevertheless, especially when employing Tamm-Dancoff truncations, or other parity vio-

[10] In practice one preferably calculates the form factor between the two lightest mesons since for those the numerical convergence is fastest).

[11] In this limit, zero-mode corrections are expected only for bad currents!

lating (un-extrapolated) cutoffs, one must consider modifying even the good current operator from its canonical form. A detailed discussion of this would go beyond the intended scope of this paper, but it should be emphasized that parity conditions might also be helpful in a self-consistent determination of the current operator in those cases.

IV. A NON-PERTURBATIVE EXAMPLE

The most simple example, where the issue of parity invariance and LF quantization can be studied, is the Yukawa model in 1+1 dimensions

$$\mathcal{L} = \bar{\psi}\left(i \not\partial - M - g\phi\right)\psi + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2, \qquad (4.1)$$

where ϕ is some scalar field. The LF quantization of this model has been studied in Refs. [25–29]. With $\psi_{(\pm)}$ as defined above, the spinor part of the above Lagrangian (4.1) reads [26]

$$\begin{aligned}\mathcal{L}_\psi &\equiv \bar{\psi}\left(i \not\partial - M - g\phi\right) \\ &= \sqrt{2}\left[\psi^\dagger_{(+)} i\partial_+\psi_{(+)} + \psi^\dagger_{(-)} i\partial_-\psi_{(-)}\right] \\ &\quad - (M+g\phi)\left[\psi^\dagger_{(+)}\gamma^0\psi_{(-)} + \psi^\dagger_{(-)}\gamma^0\psi_{(+)}\right] \qquad (4.2)\end{aligned}$$

and $\psi_{(-)}$ satisfies the constraint equation

$$\psi_{(-)} = -\frac{i}{\sqrt{2}\partial_-}\left(M+g\phi\right)\gamma^0\psi_{(+)}. \qquad (4.3)$$

Upon inserting the solution to this constraint equation (4.3) into $\mathcal{L}_\psi$ (4.2) one finds

$$\begin{aligned}\mathcal{L}_\psi &= \sqrt{2}\psi^\dagger_{(+)} i\partial_+\psi_{(+)} + \frac{M^2}{\sqrt{2}}\psi^\dagger_{(+)}\frac{i}{\partial_-}\psi_{(+)} \qquad (4.4)\\ &\quad + \frac{Mg}{\sqrt{2}}\psi^\dagger_{(+)}\left[\frac{i}{\partial_-}\phi + \phi\frac{i}{\partial_-}\right]\psi_{(+)} + \frac{g^2}{\sqrt{2}}\psi^\dagger_{(+)}\phi\frac{i}{\partial_-}\phi\psi_{(+)},\end{aligned}$$

which contains only dynamical degrees of freedom and can be quantized straightforwardly. One thus obtains the *canonical* LF Hamiltonian

$$P^-_{can} = \int dx^- \mathcal{H}_{can}, \qquad (4.5)$$

where

$$\begin{aligned}\mathcal{H}_{can} &= \frac{m^2}{2}\phi^2 - \frac{M^2}{\sqrt{2}}\psi^\dagger_{(+)}\frac{i}{\partial_-}\psi_{(+)} \qquad (4.6)\\ &\quad - \frac{Mg}{\sqrt{2}}\psi^\dagger_{(+)}\left[\frac{i}{\partial_-}\phi + \phi\frac{i}{\partial_-}\right]\psi_{(+)} - \frac{g^2}{\sqrt{2}}\psi^\dagger_{(+)}\phi\frac{i}{\partial_-}\phi\psi_{(+)}.\end{aligned}$$

The canonical Hamiltonian (4.5) contains 4 terms: a two point function for both fermions and bosons, a three point function and a four point function. From the point of view of renormalization it is thus natural to make the following ansatz for the renormalized Hamiltonian density[12]

$$\begin{aligned}\mathcal{H}_{ren} &= \frac{m^2}{2}\phi^2 - \frac{M^2}{\sqrt{2}}\psi^\dagger_{(+)}\frac{i}{\partial_-}\psi_{(+)} + \mathcal{H}_{n.o.} \qquad (4.7)\\ &\quad - \frac{c_3}{\sqrt{2}}\psi^\dagger_{(+)}\left[\frac{i}{\partial_-}\phi + \phi\frac{i}{\partial_-}\right]\psi_{(+)} - \frac{c_4}{\sqrt{2}}\psi^\dagger_{(+)}\phi\frac{i}{\partial_-}\phi\psi_{(+)}.\end{aligned}$$

The canonical Hamiltonian is obtained by taking [25]

$$c_3 = \sqrt{M^2 c_4} \qquad \text{(canonical Hamiltonian).} \qquad (4.8)$$

In perturbation theory with $\mathcal{H}_{can}$, infinities in the longitudinal momentum integral occur only at the one-loop level (for both fermion and boson self-energies) and are calculable [25]. The corresponding counter term (whose infinite part is unique) is denoted by the "normal ordering term" $\mathcal{H}_{n.o.}$. The finite part of $\mathcal{H}_{n.o.}$ has the operator structure of kinetic terms. Since such operators are already explicitly included in the above ansatz [Eq.(4.7)], it is not necessary to discuss the finite part of $\mathcal{H}_{n.o.}$ here any further.

The Lagrangian as well as the canonical Hamiltonian (4.5) contain only 3 free parameters:M, m, g. Therefore the most general situation for the Yukawa model should thus be described by fixing only 3 parameters as well. On the other hand, it is known [16,15] that the above relation (4.8) is not valid after renormalization, i.e. it seems that all 4 parameters in Eq.(4.7) get renormalized independently. However, the 4 parameters in Eq.(4.7) are not really independent! The point is that arbitrary values for the parameters m^2, M^2, c_2, c_4 do *not* correspond to the Yukawa model but rather something else. Only for a 3-dimensional subspace of the 4-dimensional parameter space spanned by m^2, M^2, c_2, c_4 does Eq.(4.7) actually describe a version of the Yukawa model. At the tree level, this 3-dimensional subspace is characterized by the canonical relation (4.8). Beyond the tree level, the relation between the 4 parameters, for which Eq.(4.7) describes a Yukawa model, is in general more complex. Hence the crucial question is: how can one find that relation? One possibility (at least in principle) is that one makes calculations both using equal time quantization as well as LF quantization. One then calculates 4 physical quantities in both schemes and fine-tunes the 4 parameters in the LF calculation until the 4 physical quantities have been reproduced. Even though there is nothing fundamentally wrong with this procedure, it is very unattractive since it requires one to go back to an equal time quantized theory in order to define the LF theory.

[12]In fact, in perturbation theory, it is both necessary but also sufficient to generalize the Hamiltonian as in Eq.(4.7) [16].

A much more satisfying approach is based on the observation that, in general, a "wrong" combination of the 4 parameters m^2, M^2, c_2, c_4 leads to a parity violating theory! [13] One can exploit this fact by means of the following renormalization procedure: first one picks (or fixes, using some at this point unspecified renormalization condition) three of the above four parameters. Then one fine tunes the fourth parameter until some parity sensitive observable indicates no violation. As a consistency condition one can check more than one parity sensitive observable.

In the application to the Yukawa model we used a variation of this generic procedure. First, the four point coupling c_4 is assigned the value 2π. [14] This only determines the overall mass scale. A completely equivalent approach would have been not to fix c_4 at all but to measure all dimensionful parameters and physical quantities in units of $\lambda \equiv c_4/2\pi$ (which carries the dimension mass squared).

Then arbitrary values for the physical masses of the lightest fermion as well as the lightest scalar meson (strictly speaking, the $C = 1$ meson, which is supposed to be scalar!) were selected. Including the condition $c_4 = 2\pi$, this implies 3 conditions and we can now start the actual fine tuning procedure which allows to fix all four bare parameters: For given values of M_F^{phys} and M_S^{phys} an arbitrary value for the vertex mass (i.e. for c_3) was selected. Then the bare masses of both fermion and boson were fine tuned so that the physical masses of both fermion and boson equal M_F^{phys} and M_S^{phys}. After this had been achieved, the inelastic transition form factor between the two lightest mesons was calculated as described above. This procedure was repeated for different values of c_3 until the inelastic form factor satisfied the parity condition discussed in section III.

In the numerical procedure, DLCQ [25], with anti-periodic boundary conditions for the fermions and periodic boundary conditions for the bosons, was employed. No cutoff beyond the DLCQ cutoff was used, i.e. particle number was allowed to reach arbitrarily large values — the only limit was set by the DLCQ parameter K, which measures the total momentum of the initial meson in the transition matrix element [Eq.(3.2)] in units of $2\pi/L$, where L is the box length in DLCQ. The DLCQ-cutoff itself violates parity. It is therefore necessary, to verify that the results have numerically converged in K. The matrix elements were calculated for $K = 24, 32$, and 40. Typical results are shown in Figs. 1 and 2. Fig. 1 corresponds to intermediate coupling $[\left(M_F^{phys}\right)^2 = \left(M_F^{phys}\right)^2 = 4$ in units of the coupling constant], while Fig.2 $[\left(M_F^{phys}\right)^2 = \left(M_F^{phys}\right)^2 = 2]$ is an example for strong coupling.

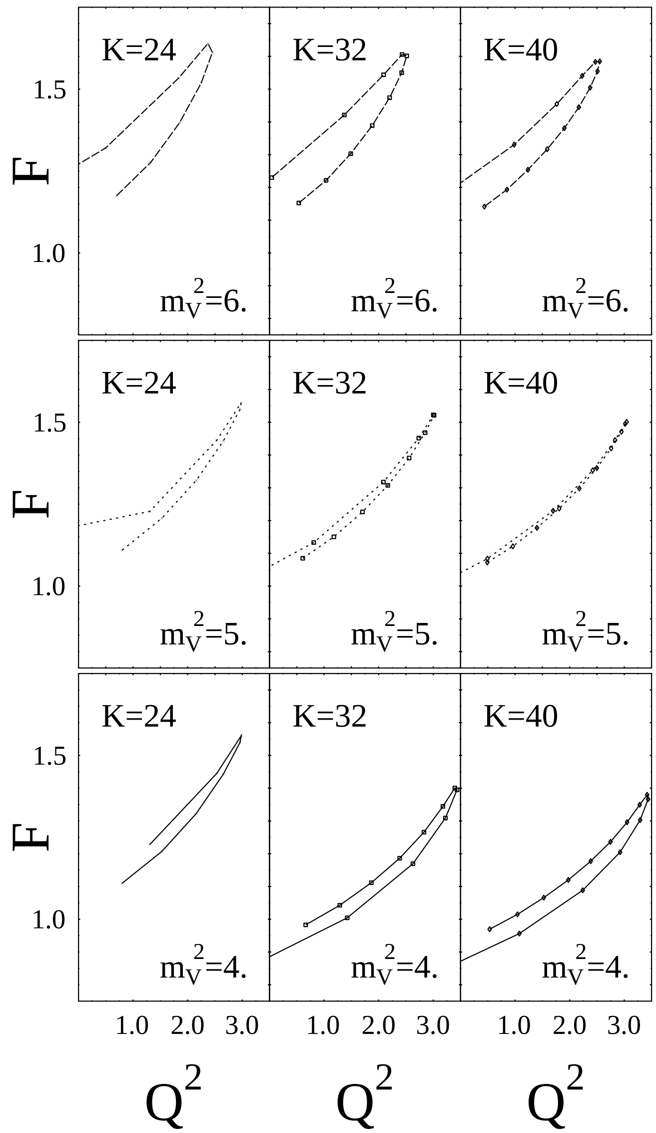

FIG. 1. Inelastic transition form factor (3.2) between the two lightest meson states of the Yukawa model, calculated for various vertex masses m_v and for various DLCQ parameters K. The physical masses for the fermion and the scalar meson have been renormalized to the values $\left(m_F^{phys}\right)^2 = \left(m_F^{phys}\right)^2 = 4$. All masses and momenta are in units of $\sqrt{\lambda} = \sqrt{c_4/2\pi}$.

In the calculations, equal physical masses for the fermion and the scalar meson were chosen because if the fermion and meson have similar masses there is only one scale and the numerical convergence (in K) is faster. In principle,

[13] One can easily convince oneself about this fact for example by calculating some physical observables at tree level, but with a combination of parameters that does not satisfy the canonical relation [Eq.(4.8)].

[14] Note, Yukawa$_{1+1}$ is superrenormalizable and there is only a finite renormalization of the four point coupling which we ignore here for simplicity. Alternatively we could have imposed a condition on a physical observable here.

there is nothing wrong with repeating this procedure for unequal masses.

Since momenta assume only discrete values in DLCQ, the form factor could only be evaluated at a discrete set of points. For example, with an initial momentum of $K = 40$ for the "pseudoscalar" meson, the final momentum of the "scalar" meson was taken to be $K = 38, 36, 34, \ldots$. In the plots, the form factors, evaluated between states with these momenta, were connected by a smooth curve to guide the eye.

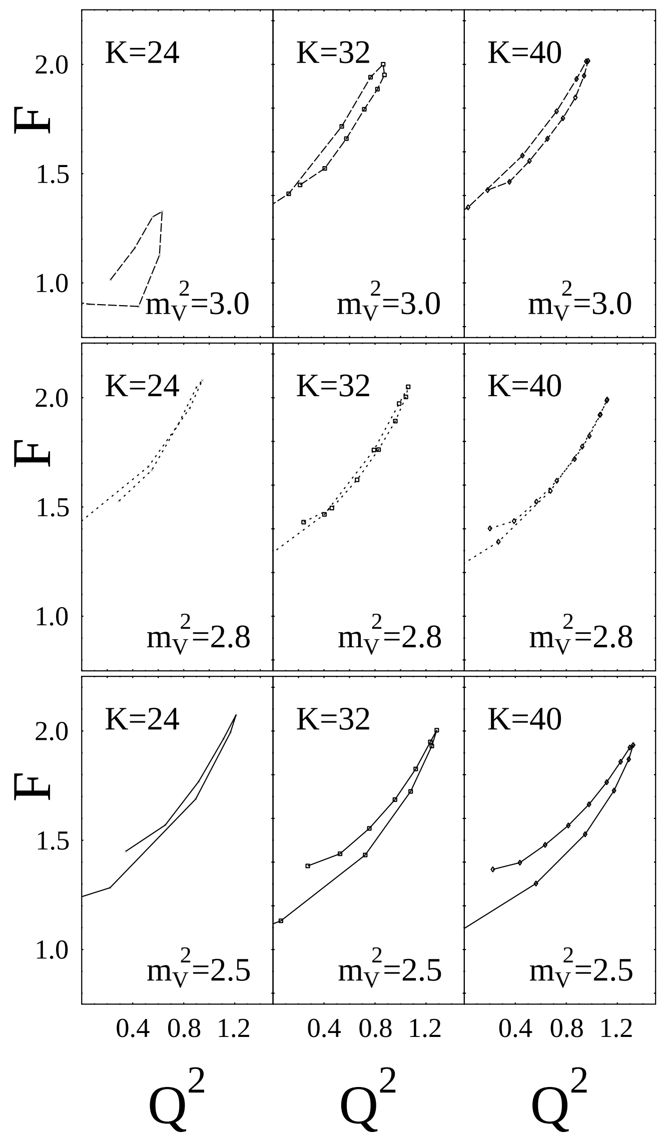

FIG. 2. Same as Fig.1 but for $\left(m_F^{phys}\right)^2 = \left(m_F^{phys}\right)^2 = 2$.

From the fact that the form factors for $K = 32$ and $K = 40$ hardly differ, one can conclude that the results have converged numerically. The examples in Figs.1 and 2 show several facts:

- For arbitrary combinations of physical masses (or bare kinetic masses) and vertex masses, the transition form factor F_{mn} (3.2) is *not* a unique function of q^2 (even for K large), thus clearly demonstrating a violation of parity in the general case.
- For specific combinations of physical masses (or bare kinetic masses) and vertex masses, the transition form factor F_{mn} (3.2) is a unique function of q^2. For given values for the physical masses, a unique vertex mass, which renders the form factor parity invariant, was found: In the case of $\left(m_F^{phys}\right)^2 = \left(m_F^{phys}\right)^2 = 4$, the correct value for the vertex mass is about $m_V^2 \approx 5$, while for $\left(m_F^{phys}\right)^2 = \left(m_F^{phys}\right)^2 = 2$ the correct value is near $m_V^2 \approx 2.8$.
- It should be emphasized that the transition form factor is a function and not just one number, i.e. the mere fact that F_{mn} is parity invariant over the whole range of q^2 considered provides a consistency check of the procedure described in this work. This fact, plus the uniqueness mentioned above, give a strong indication that we have really found the correct renormalized Hamiltonian and that the procedure outlined in this paper is practical.

As a side remark, it should be explained here how the bare masses were fixed in practice. There are two slightly different procedures one can imagine. In the first procedure one first adds a momentum dependent kinetic term that takes care of the one loop divergences in the self-energies. Then one adds a finite (momentum independent) bare masses for boson and fermion which are fine tuned until the physical masses take the values desired in the large K limit. In this work, a slightly different procedure was used: momentum dependent bare masses for both boson and fermion were introduced in such a way (by fine-tuning) that the physical masses are K independent. This can be easily done in a successive procedure. In the large K limit, the momentum dependence of the kinetic term thus obtained reproduces the momentum dependence as derived from the one loop counter term and therefore both procedures agree with each other in the large K limit. However, it was found that, typically, when the physical masses of the lightest particles are exactly K independent (and not only in the limit $K \to \infty$), other physical observables converge faster to their $K \to \infty$ values.

V. SUMMARY

We have investigated several classes of observables, which are potentially sensitive to parity violations, and found that most of them are *not* suitable for "typical" LF calculations. What makes them "unsuitable" is that they involve scattering states (which are notoriously difficult for most non-perturbative numerical algorithms) or

they involve matrix elements of "bad" currents (which are often ill defined in the LF formalism). Other observables, which seem *a priory* sensitive to parity violations of the formalism are often "protected" by manifest LF symmetries, such as C-parity.

We found one observable which is both sensitive to parity violations and easily accessible in a standard LF calculation: inelastic matrix elements of the good component of the current operator. Vector current conservation demands

$$q^- \langle m, p'|j^+(0)|n, p\rangle + q^+ \langle m, p'|j^-(0)|n, p\rangle = 0, \quad (5.1)$$

while parity invariance implies

$$\langle m, p'|j^-(0)|n, p\rangle = \mp\langle m, \bar{p'}|j^+(0)|n, \bar{p}\rangle, \quad (5.2)$$

where $\bar{p}^\pm \equiv p^\mp = M_n^2/2p^\pm$, $\bar{p'}^\pm \equiv p'^\mp = M_m^2/2p'^\pm$ are the parity transformed momenta of the initial and final state and the sign in Eq.(5.2) depends on whether the two states m and n have the same or opposite intrinsic parity. Eqs.(5.1) and (5.2) individually are useless, since they involve a bad current matrix element. However, notice that the bad current matrix element in Eq. (5.1) and in Eq. (5.2) is the same. The trick is to use Eq. (5.2) to eliminate the matrix element of j^- from Eq.(5.1) and one obtains a relation between two matrix elements of the good current at different values of q^+ but at the same value of q^2, yielding

$$q^- \langle m, p'|j^+(0)|n, p\rangle = \pm q^+ \langle m, \bar{p'}|j^+(0)|n, \bar{p}\rangle. \quad (5.3)$$

We demonstrated explicitly that the parity relation thus obtained (5.3) is useful for practical calculations by applying it to a concrete non-perturbative example: Yukawa$_{1+1}$. In the LF formulation, the renormalized Hamiltonian contains one more "free" parameter than the Lagrangian. Since the additional counter term involved is not parity invariant one can use the parity relation derived in this paper as an additional renormalization condition. We studied a few cases numerically and showed that the fine tuning procedure can be done also in practice. Since the parity relation for the inelastic transition form factor is in fact not just one relation, but an infinite number (it involves functions!), we obtained at the same time a strong self consistency check for the whole procedure.

Can a similar procedure be applied to help in constructing the LF-Hamiltonian for QCD? First of all, since the parity relation derived in this paper involves only meson states, color singletts and physical (gauge invariant) operators, there is nothing special about QCD. Even though QCD is a gauge theory and even though one usually picks the $A^+ = 0$ (or similar) condition on the gauge field, the parity relation for the good current should still hold if the LF formulation is to be parity invariant at the level of physical observables. However, some conditions must be satisfied before one can do this in practice: Either one must be sure that all cutoffs that violate parity, such as Tamm-Dancoff truncations of longitudinal momentum cutoffs or energy cutoffs must be taken large (or small) enough so that they no longer affect the states under consideration. This is hard, but not impossible! For example, on a small transverse lattice, one can get rather close to the longitudinal continuum limit in practical calculations. However, in examples were one cannot remove those cutoffs, one must carefully renormalize the currents before calculating the necessary matrix elements.

Even though we focused on inelastic transition matrix elements of the good component of the vector current, there are more "useful" parity relations beyond the inelastic vector transition matrix element. For example, there exist also some useful "parity-relations" among virtual Compton amplitudes which may also be useful in the context of QCD. However, a detailed investigation of such observables will be postponed to some forthcoming paper.

In any case, parity relations such as the ones derived in this paper should be useful in the search for P^-_{QCD}, since they imply a strong consistency check for the states that arise as solutions from diagonalizing P^-_{QCD}.

---

[1] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[2] K. Hornbostel, in "From Fundamental Fields to Nuclear Phenomena", Eds. J. A. McNeil and C. E. Price, (World Scientific, Singapore, 1991).
[3] K. G. Wilson et al., Phys. Rev. D **49**, 6720 (1994).
[4] K. G. Wilson and M. Brisudova, in *Theory of Hadrons and Light-Front QCD*, Proceedings, Zakopane, 1994, hep-th/9411008.
[5] K. G. Wilson and D. Robertson, in *Theory of Hadrons and Light-Front QCD*, Proceedings, Zakopane, 1994, hep-th/9411007.
[6] S. J. Brodsky and D. G. Robertson, talk presented at the ELFE meeting on Confinement Physics, Cambridge, Eng., July 1995, hep-ph/9511374.
[7] A. Harindranath and J. Vary, Phys. Rev. D **36**, 1141 (1987).
[8] A. Harindranath and J. Vary, Phys. Rev. D **37**, 1064 (1988).
[9] F. Lenz, S. Levit, M. Thies and K. Yazaki, Ann. Phys. (N.Y.) **208**, 1 (1991); F. Lenz, Proceedings of NATO Advanced Study Institute on "Hadrons and Hadronic Matter", Eds. Vautherin et al. (Plenum, New York, 1990).
[10] K. Hornbostel, Phys. Rev. D **45**, 3781 (1992).
[11] D. G. Robertson, Phys. Rev. D **47**, 2549 (1993).
[12] M. Burkardt, Phys. Rev. D **47**, 4628 (1993).
[13] S. S. Pinsky and B. vande Sande, Phys. Rev. D **48**, 816 (1993);**49**, 2001 (1994); S. S. Pinsky, B. vande Sande and J. R. Hiller, Phys. Rev. D **51**, 762 (1995).
[14] E. V. Prokhvatilov and V. A. Franke, Sov. J. Nucl. Phys. **49**, 688 (1989); E. V. Prokhvatilov, H. W. L. Naus and H.-J. Pirner, Phys. Rev. D **51**, 2933 (1995); J.P. Vary

and T.J. Fields, talk given at 4th International Workshop on "Light Cone Quantization and Non-Perturbative Dynamics, Polona Zgorzelisko", ed. S. Glazek, Poland, 15-25 Aug 1994, hep-ph/9411263.
[15] M. Burkardt, Adv. Nucl. Phys. **23**, 1 (1996).
[16] M. Burkardt and A. Langnau, Phys. Rev. D **44**, 3857 (1991).
[17] W. A. Bardeen and R. B. Pearson, Phys. Rev. D **14**, 547 (1976); W. A. Bardeen, R. B. Pearson and E. Rabinovici, **21**, 1037 (1980).
[18] P. A. Griffin, Mod. Phys. Lett. A **7**, 601 (1992); P. A. Griffin, Nucl. Phys. B **372**, 270 (1992).
[19] P. A. Griffin, Phys. Rev. D **47**, 1530 (1993).
[20] M. Burkardt, talk presented at ELFE Meeting on Confinement Physics, Cambridge, Eng., July 1995, hep-ph/9510264.
[21] M. Burkardt, *submitted to Phys. Rev. D* , hep-ph/9512318.
[22] M. Burkardt, Phys. Lett. B **268**, 419 (1991); Nucl. Phys. B **373**, 613 (1992); Phys. Rev. D **52**, 3841 (1995); W.-M. Zhang, Chin. J. Phys. **32**, 717 (1994).
[23] J. Hiller, Phys. Rev. D **43**, 2418 (1991).
[24] M. Burkardt, Phys. Rev. D **49**, 5446 (1994).
[25] H.-C. Pauli and S. J. Brodsky, Phys. Rev. D **32**, 1993 (1985); 2001 (1985).
[26] S.-J. Chang and T.-M. Yan, Phys. Rev. D **7**, 1147 (1972).
[27] A. Harindranath and R. J. Perry, Phys. Rev. D **43**, 492 (1991); *erratum* 3580 (1991).
[28] D. G. Robertson and G. McCartor, Z. Phys. C **53**, 661 (1992)
[29] S. D. Glazek et al., Phys. Rev. D **47**, 1599 (1993).
[30] R. J. Perry, invited lectures presented at 'Hadrons 94', Gramado, Brazil, April 1994, hep-th/9407056.